Defense Technical Information Center Compilation Part Notice

ADP012496

TITLE: Two-Stream Instabilities in Guiding-Center Plasmas or Antihydrogen Recombination Schemes

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: Non-Neutral Plasma Physics 4. Workshop on Non-Neutral Plasmas [2001] Held in San Diego, California on 30 July-2 August 2001

To order the complete compilation report, use: ADA404831

The following component part numbers comprise the compilation report:
ADP012489 thru ADP012577

Two-Stream Instabilities in Guiding-Center Plasmas for Antihydrogen Recombination Schemes [1]

Ronald Stowell and Ronald C. Davidson

Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543

Abstract. Two-stream instabilities are studied analytically in the guiding-center kinetic regime, which is chosen in order that the results may be applied to the mixing of antiprotons and positrons preceding antihydrogen recombination. The guiding-center kinetic description is valid for a range of parameters which includes cases in which magnetic fields are $3-5\,\mathrm{T}$; temperatures are $4-10\,\mathrm{K}$; positron densities are $10^7-10^8\,\mathrm{cm}^{-3}$; and antiproton densities are $10^4-2\times10^7\,\mathrm{cm}^{-3}$. The species occupy long, cylindrical columns coaxial with an outer, conducting, cylindrical wall. Plasma column radii are between 0.05 and 0.3 wall radii. A constant, axial, externally generated magnetic field permeates the system. Linear stability of the plasma is studied as a function of the species' temperatures, densities and column radii; the mean antiproton velocity; and wavenumber. Drifting Maxwellian distribution functions are considered.

INTRODUCTION

For fifteen years, cold, stationary antihydrogen has been sought [1, 2] for use in fundamental experiments, including tests of the equivalence principle of general relativity [1] and the CPT symmetry of the standard model [3]. The protracted nature of this effort is the result of technical challenges, such as deceleration and cooling of the constituent species [4, 5] and, nonetheless, exceedingly slow recombination rates [6]. Only recently have cold antiprotons and positrons been allowed to interact, though no attempt has been made yet to detect recombined antihydrogen [7].

As the experimentalists successfully accumulate antimatter, collective descriptions of the interacting species will become more useful. Accordingly, plasma phenomena with which we are familiar in other contexts may appear. In particular, two-stream instabilities, manifest as the electron-proton (e-p) instability in proton storage rings [8], could occur in antihydrogen traps when the constituent species are mixed. There is free energy to drive this instability because the system in not in thermal equilibrium during mixing. The following describes preliminary research toward assessing the possibility of two-stream instabilities in cold antimatter plasmas and finding the regimes of stable operation of the traps.

[1] Research supported by the United States Department of Energy, and in part by the Office of Naval Research.

CP606, *Non-Neutral Plasma Physics IV*, edited by F. Anderegg et al.

Currently there are two distinct collaborations seeking to synthesize cold antihydrogen at rest in the laboratory frame. Each group has its own scheme for inducing recombination. The ATHENA collaboration is considering a method described as follows. Antiprotons are sent into a cloud of positrons where the antiprotons are cooled by collisions with the positrons [9]. Escaping antiprotons would be caught electrostatically and sent back into the positron cloud. The ATRAP collaboration releases bunches of cold antiprotons from an electrostatic well into a positron cloud [7]. As long as ATHENA's antiproton densities are extremely low, a plasma description is not appropriate. However, this will change as ATHENA will ultimately seek to increase production of antihydrogen. A plasma model is already useful for ATRAP's parameters.

GENERAL THEORETICAL MODEL

It may be worrisome that the coupling parameter $\Gamma = \left[e^2/(4\pi n/3)^{-1/3}\right]/k_BT$ is only one or two orders of magnitude smaller than unity. Even a modest improvement in densities or temperatures will force the system into the regime of a Wigner liquid. However, for the parameters presently envisioned, a plasma description of the antimatter is valid.

The geometry considered is shown in Fig. 1. Two cylindrical, coaxial plasma columns of equal radius, composed of positrons and antiprotons, are coaxial with an outer, cylindrical, conducting wall. The antiprotons move through the cloud of positrons which are stationary on average. Some important approximations which are made to solve the problem mathematically are as follows. Bunches, or clouds, of particles of finite extent are treated as having infinite length. This is a valid approximation if we study the system on a scale much smaller than the bunch length. For the present purposes, the bunches are also treated as having the same radius, uniform density and Maxwellian unperturbed distribution functions f_{s0}. Our attention is restricted to azimuthally symmetric perturbed distribution functions f_{s1}. The externally generated part ϕ^{ext} of the electrostatic potential ϕ is assumed to be constant in space and time. Collisions are not included in the analysis.

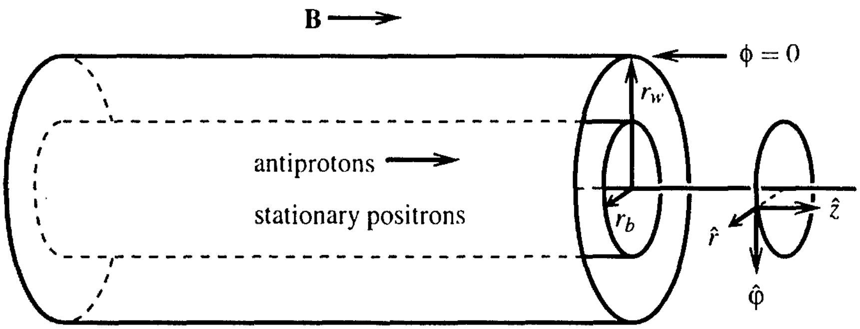

FIGURE 1. Geometry Considered.

From the values of the antihydrogen trap parameters in Nonneutral Plasma Physics

III [2] [10, 11, 12, 13], it can be seen that, for relevant temporal and spatial scales, the antimatter plasmas fall within the gyrokinetic regime, which is, by definition, the regime in which $\omega/\omega_{c\bar{p}}$, $(e\phi/T_{e^+})(k_\perp\rho_{L\bar{p}})^2$, $k_\parallel/k_\perp$ and $\rho_{Ls}/L_\perp$ are small quantities [14, 15, 16]. Here ω is any frequency considered, ω_{cs} is the cyclotron frequency for species s, ϕ is the electrostatic potential, T_s is the temperature of species s, $\mathbf{k}$ is any wavenumber considered, ρ_{Ls} is the Larmor radius for species s and $L_\perp$ is the perpendicular length of the system. The subscripts "$\perp$" and "$\parallel$" indicate components perpendicular and parallel to the magnetic field $\mathbf{B}$, respectively. Furthermore, the plasmas are in the drift kinetic regime, which is, by definition, that part of the gyrokinetic regime in which $k_\perp\rho_{L\bar{p}} \ll 1$. The plasmas are in the guiding-center kinetic regime, which is, by definition, that part of the drift kinetic regime in which $\omega_{ps}^2/\omega_{cs}^2 \ll 1$ for each species, where ω_{cs} and ω_{ps} are the cyclotron and plasma frequencies, respectively, for species s.

From $\omega/\omega_{c\bar{p}} \ll 1$, it follows that the $\mathbf{E}\times\mathbf{B}$ drift is much greater than the polarization drift. Since $T_{e^+}/e\phi \ll 1$, where ϕ is the electrostatic potential, the $\mathbf{E}\times\mathbf{B}$ drift is much greater than the curvature and ∇B drifts. The Banös drift [17, 18] is extremely small.

If the longitudinal and transverse waves are decoupled, we may further restrict attention to the electrostatic approximation,

$$\nabla\cdot\mathbf{E} = -\nabla^2(\phi^{\text{self}}+\phi^{\text{ext}}) = 4\pi \sum_{s\in\{e^+,\bar{p}\}} q_s \int d^3\mathbf{v} f_s. \tag{1}$$

Here ϕ^{self} and ϕ^{ext} are the parts of the electrostatic potential due to the plasma and external sources, respectively, q_s is the charge of species s, $f_s(\mathbf{r},\mathbf{v},t)$ is the distribution function for species s and $\mathbf{v}$ is velocity. Since, for each species, the collision frequencies $\nu_{ss'}$ are small compared to the plasma frequencies ω_{ps}, collisions may be neglected.

The above approximations lead to the collisionless guiding-center kinetic equation

$$\frac{\partial f_s}{\partial t} + \left[\frac{c\mathbf{E}\times\mathbf{B}}{B^2}\right]\cdot\nabla_\perp f_s + v_\parallel\nabla_\parallel f_s + \frac{q_s}{m_s}E_\parallel\frac{\partial f_s}{\partial v_\parallel} = 0. \tag{2}$$

The distribution functions f_s have units of volume$^{-1}\cdot$velocity^{-3}. The quantity m_s is the mass of species s. In cylindrical coordinates (r,φ,z), with a constant, axial magnetic field $\mathbf{B} = B\hat{z}$ and ϕ^{ext} independent of position and time, Eqs. (1) and (2) may be simplified. Perturbing the result about an equilibrium solution $(f_{e^+0}, f_{\bar{p}0}, \phi_0)$ which is independent of φ and z and applying the Fourier-Laplace transform $(F_s,\Phi) = (2\pi)^{-2}\int_{-\infty}^{\infty}dz\int_0^{2\pi}d\varphi\int_0^\infty dt\, e^{-i(\ell\varphi+k_z z-\omega t)}(f_{s1},\phi_1^{\text{self}})$, where $\text{Im}[\omega]>0$, to the three first-order equations for the perturbed solution $(f_{e^+1}, f_{\bar{p}1}, \phi_1)$ yields an inhomogeneous initial-value problem. The corresponding homogeneous equation is

$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \frac{\ell^2}{r^2} - k_z^2 - \sum_s \frac{\omega_{ps}^2}{n_{s0}}\int d^3\mathbf{v}\frac{k_z\frac{\partial f_{s0}}{\partial v_z} + \frac{1}{\omega_{cs}}\frac{\ell}{r}\frac{\partial f_{s0}}{\partial r}}{\omega - \ell\omega_E(r) - k_z v_z}\right)\Phi = 0, \tag{3}$$

[2] The radius of ATRAP's positron bunch was estimated from Fig. 1 of reference [10] to be 0.15 mm. The radius and length of ATHENA's antiproton bunch and the radius of ATRAP's antiproton bunch were estimated from ASACUSA's data [11] to be 0.1 cm, 5.0 cm and 0.1 cm, respectively. All other data appears numerically in the appropriate articles [10, 12, 13].

where $\omega_E = -cE_0(r)/rB_0$ is the equilibrium $\mathbf{E}\times\mathbf{B}$ rotation frequency. Here the subscripts 0 and 1 indicate unperturbed equilibrium quantities and perturbed quantities, respectively. The boundary conditions are that Φ is zero at the wall, finite on the trap's axis and continuous at the plasma edge. The dispersion relation is obtained using the additional condition found by integrating Eq. (3) across the plasma edge.

APPLICATION TO COLD PLASMAS

By "cold," it is meant that both parallel thermal velocities are small compared to the average parallel velocity of the antiprotons. In this special case, which is a simple place to begin,

$$f_{s0} = n_{s0} H(r_b - r)\, G(\mathrm{v}_\perp)\, \delta(\mathrm{v}_z - \mathrm{u}_{sz0}). \tag{4}$$

Here n_{s0} is the equilibrium density, which is constant in space; H is the Heaviside step function; r_b is the plasma radius, which is the same for both species; and $G(\mathrm{v}_\perp)$ is an arbitrary distribution in perpendicular velocity space normalized according to $2\pi\int_0^\infty d\mathrm{v}_\perp \mathrm{v}_\perp G(\mathrm{v}_\perp) = 1$. If (4) is used with $\ell = 0$, Eq. (3) may be simplified to

$$1 = \frac{\mathrm{k}_z^2}{\mathrm{k}_r^2 + \mathrm{k}_z^2} \sum_s \frac{\omega_{ps}^2}{(\omega - \mathrm{k}_z \mathrm{u}_{sz0})^2}, \tag{5}$$

where k_r solves

$$\mathrm{k}_z \frac{K_0(\mathrm{k}_z r_w) I_1(\mathrm{k}_z r_b) + K_1(\mathrm{k}_z r_b) I_0(\mathrm{k}_z r_w)}{K_0(\mathrm{k}_z r_w) I_0(\mathrm{k}_z r_b) - K_0(\mathrm{k}_z r_b) I_0(\mathrm{k}_z r_w)} + \mathrm{k}_r \frac{J_1(\mathrm{k}_r r_b)}{J_0(\mathrm{k}_r r_b)} = 0. \tag{6}$$

Here, $\mathbf{u}_s$ is the average velocity of species s; r_w is the wall radius; and I_n, J_n and K_n are Bessel functions. The transcendental equation (6) may be solved for $\mathrm{k}_r(\mathrm{k}_z)$, which, for any particular value of k_z, is quantized with a radial mode number n. The solutions for k_r, which are displayed in Fig. 2, are then substituted into (5).

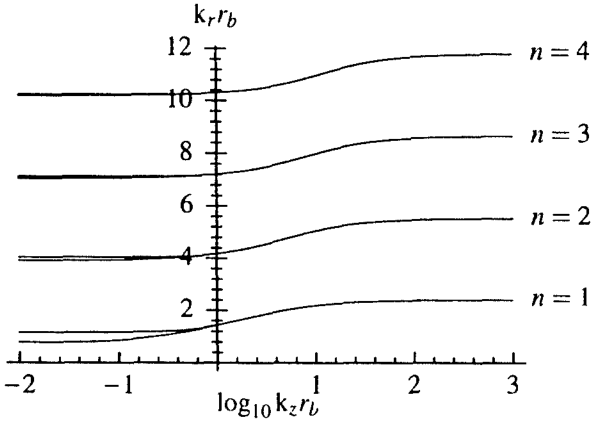

FIGURE 2. Solutions to (6). For a given radial mode number n, the upper and lower branches are for $r_b/r_w = 0.3$ and 0.05, respectively, which bound the range of values of r_b/r_w of practical interest.

Using *Mathematica*®, the quartic equation (5) for ω was solved both analytically and numerically with the same results. Eq. (5) has four roots. One root is unstable for a range of values of k_z. The unstable root, for physical values of the trap parameters, is shown in Fig. 3. The oscillation frequency $\mathrm{Re}[\omega]/\omega_{pe^+}$ and the growth rate $\mathrm{Im}[\omega]/\omega_{pe^+}$ are displayed as functions of $k_z u_{\bar{p}z0}/\omega_{pe^+}$. As expected [19], for a cold plasma, the maximum growth rate scales as $\omega_{pe^+}^{1/3}\omega_{p\bar{p}}^{2/3}$.

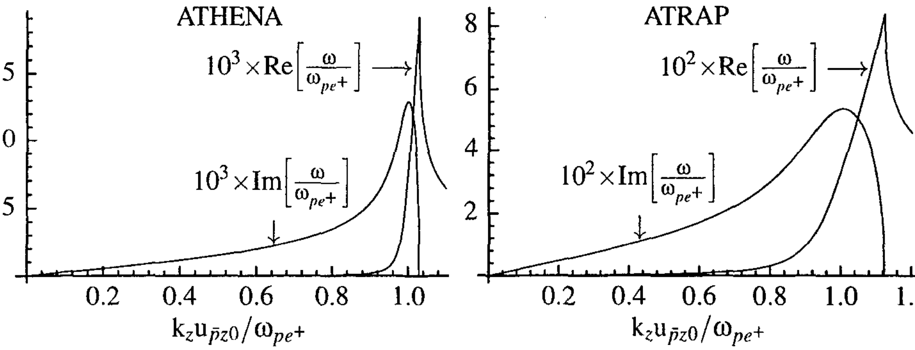

FIGURE 3. The unstable root of (5) for $n = 1$. The left and right plots are for ATHENA and ATRAP, respectively; r_b/r_w is 0.1 and 0.125, respectively; $\omega_{pe^+}/\omega_{p\bar{p}}$ is 380 and 44, respectively; and $u_{\bar{p}z0}/(\omega_{pe^+} r_b)$ is 5.8×10^{-4} and 8.4×10^{-4}, respectively.

APPLICATION TO DRIFTING MAXWELLIAN PLASMAS

Now the unperturbed distribution functions are allowed to have nonzero parallel thermal velocities v_{tsz}:

$$f_{s0} = \frac{n_{s0} H(r_b - r)}{v_{tsz}\sqrt{2\pi}} G(v_\perp) \exp\left[-\frac{(v_z - u_{sz0})^2}{2v_{tsz}^2}\right]. \tag{7}$$

If Eq. (7) is substituted into Eq. (3) with $\ell = 0$, Eq. (3) takes the form

$$-\sum_s \frac{1}{\lambda_{Ds}^2}\left(1 + i\sqrt{\pi}\frac{\omega - k_z u_{sz0}}{k_z v_{tsz}\sqrt{2}} \exp\left[-\left(\frac{\omega - k_z u_{sz0}}{k_z v_{tsz}\sqrt{2}}\right)^2\right]\left(1 + \mathrm{erf}\left[i\frac{\omega - k_z u_{sz0}}{k_z v_{tsz}\sqrt{2}}\right]\right)\right)$$

$$= k_r^2 + k_z^2, \tag{8}$$

where $\lambda_{Ds} = v_{tsz}/\omega_{ps}$ is the Debye length for species s, $\mathrm{erf}(z) = (2/\sqrt{\pi})\int_0^z dt\, e^{-t^2}$ and k_r solves (6), exactly as for a cold plasma.

Using *Mathematica*®, the transcendental equation (6) for $k_r(k_z)$ was solved numerically, and the result was substituted into the transcendental equation (8), which was also solved numerically. For ATHENA's and ATRAP's parameters, it is found that there is a critical average velocity for the antiprotons below which the system is stable. In these cases and in all others examined, it was found that the critical velocity of the antiprotons is the positron thermal velocity times a number of order unity. For both ATHENA's

and ATRAP's parameters, typical maximum growth rates are found to be of the order of $\omega_{pe^+}^{1/3}\omega_{p\bar{p}}^{2/3}$ when $u_{\bar{p}z0}/v_{tsz} \gg 1$.

In the second section, terms scaling like the collision frequencies were neglected in favor of terms scaling like the plasma frequencies. However, the growth rates can be comparable in size to $\omega_{pe^+}^{1/3}\omega_{p\bar{p}}^{2/3}$, which is about the same size as the largest collision frequency, the positron-positron collision frequency. This suggests that collisions play an important role in the damping or growth of the mode. Fortunately, to leading order, positron-positron collisions are described by O'Neil's collision operator [20, 21]. Inclusion of collisions is a logical next step in the calculations.

REFERENCES

1. Gabrielse, G., Fei, X., Helmerson, K., Rolston, S., Tjoelker, R., Trainor, T., Kalinowsky, H., Haas, J., and Kells, W., *Physical Review Letters*, **57**, 2504–2507 (1986).
2. Holzscheiter, M., and Charlton, M., *Reports on Progress in Physics*, **62**, 1–60 (1999).
3. Gabrielse, G., Fei, X., Haarsma, L., Rolston, S., Tjoelker, R., Trainor, T., Kalinowsky, H., Haas, J., and Kells, W., *Physica Scripta*, **T**, 36–40 (1988).
4. Surko, C., Greaves, R., and Charlton, M., *Hyperfine Interactions*, **109**, 181–188 (1997).
5. Lefevre, P., Mohl, D., and Plass, G., "The CERN Low Energy Antiproton Ring (LEAR) project", in *11th International Conference on High-Energy Accelerators*, Birkhauser Verlag, 1980, pp. 819–823.
6. Gabrielse, G., Rolston, S., Haarsma, L., and Kells, W., *Hyperfine Interactions*, **44**, 287–294 (1988).
7. Gabrielse, G., First positron cooling of antiprotons, and prospects for simultaneous trapping of neutrals and charged particles (2001), poster presentation at the 2001 Workshop on Nonneutral Plasmas.
8. Neuffer, D., Colton, E., Fitzgerald, D., Hardek, T., Hutson, R., Macek, R., Plum, M., Thiessen, H., and Wang, T.-S., *Nuclear Instruments and Methods in Physics Research*, **A321**, 1–12 (1992).
9. Private communication with L. V. Jørgensen (2001), on August 1.
10. Gabrielse, G., Estrada, J., Peil, S., Roach, T., Tan, J., and Yesley, P., "Progress Toward Cold Antihydrogen", in *Nonneutral Plasma Physics III* (AIP Conference Proceedings 498), edited by J. J. Bollinger, R. L. Spencer, and R. C. Davidson, American Institute of Physics, 1999, pp. 29–39.
11. Yamazaki, Y., "Trapping, Cooling and Extraction of Antiprotons, and the ASACUSA Project", in *Nonneutral Plasma Physics III* (AIP Conference Proceedings 498), edited by J. J. Bollinger, R. L. Spencer, and R. C. Davidson, American Institute of Physics, 1999, pp. 48–58.
12. Surko, C., Gilbert, S., and Greaves, R., "Progress in Creating Low-energy Positron Plasmas and Beams", in *Nonneutral Plasma Physics III* (AIP Conference Proceedings 498), edited by J. J. Bollinger, R. L. Spencer, and R. C. Davidson, American Institute of Physics, 1999, pp. 3–9.
13. Fine, K., "The ATHENA Antihydrogen Experiment", in *Nonneutral Plasma Physics III* (AIP Conference Proceedings 498), edited by J. J. Bollinger, R. L. Spencer, and R. C. Davidson, American Institute of Physics, 1999, pp. 40–47.
14. Rutherford, P., and Frieman, E., *Physics of Fluids*, **11**, 569–585 (1968).
15. Taylor, J., and Hastie, R., *Plasma Physics*, **10**, 479–494 (1968).
16. Lee, W., *Physics of Fluids*, **26**, 556–562 (1983).
17. White, R. B., *Theory of Tokamak Plasmas*, North-Holland Physics, Elsevier Science Publishers B.V., P.O. Box 103, 1000 AC Amsterdam, The Netherlands, 1989, see p.66.
18. Littlejohn, R., *Physics of Fluids*, **24**, 1730 (1981).
19. Davidson, R. C., *Physics of Nonneutral Plasmas*, Addison-Wesley Publishing Company, 350 Bridge Parkway, Suite 209, Redwood City, CA 94065, 1990, see equation (5.13.8) on page 271.
20. O'Neil, T., *Physics of Fluids*, **26**, 2128–2135 (1983).
21. Dubin, D. H., and O'Neil, T., *Physical Review Letters*, **78**, 3868–3871 (1997), see equation (8).